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A TOTAL STUDY OF Zn $^{68-70}$ (EVEN-EVEN) ISOTOPES

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Abstract:

In this research the nuclear structure for isotope($^{68-70}$Zn) have been studied using the Interaction Boson Model (IBM-1), this nucleus was determined depending on the practical available values and calculating energy levels In addition to the electric transition probabilities and potential Energy Surface using IBMP-Code in U(5)-O(6) and U(5). The results obtained from this study showed a good agreement with practical values. The current study has shown that the ^{70}Zn nucleus belongs to the vibration limit different with the ^{68}Zn, which has characteristics between vibrational limit and γ-unstable limit depending on the results of energy levels. Contour lines and axillary symmetries, which have been drown, were confirmed that there is no nuclear deformation but vibrations around the position of balance.

Keywords:

Introduction:

Using dynamical symmetries in order to describe nuclear collective motion, starting with many-boson system, s (L = 0) and d (L = 2) bosons was introduced by (Iachello and Arima) [1,2,3,4]The relation to the nuclear shell model has been studied [5]. In theoretical nuclear physics research ,These boson models have given rise to a new momentum, This interacting boson model relies heavily on group theory; in particular the unitary group U (6) structure of interacting s bosons and d boson, depending on annihilation and creation operators [6]. This model study heavy nuclear and middling mass number, which is far from the closed shells. The IBM model depends on two main principles. One-low-lying collective states in nuclei can be described by the valence nucleons, which make interacting fermion pairs. Two- to formed bosons the must fermion pair's couple, carrying angular momentum (J). For the s boson, there is one available magnetic substate, resolute by (2J + 1), and for the d boson ,five available magnetic substates, forming a six-dimensional space , described by the group structure.

Symmetries of the IBM

for many problems Specifically Nuclear Physics, when the Hamiltonian has certain symmetries , the solutions can be obtained . Rotational invariance leads to the possibility of characterizing the angular Eigen functions with quantum numbers (m and L) .These quantum numbers relate to the representation of the O(3) and O(2) rotation groups in two and three dimensions , respectively. The group structure of the (IBM) can be discussed in Hilbert space with six-dimensional. spanned by the d_μ ($-2 \le \mu \le 2$) and s^+ bosons [7] and the most general Hamiltonian is[8]:

$$\hat{H} = \varepsilon_s \left(\hat{s}^\dagger . \hat{\tilde{s}}\right) + \varepsilon_d \sum_m (\hat{d}^\dagger{}_m . \hat{\tilde{d}}_m) + \sum_{L=0,2.4} \frac{1}{2}\sqrt{2L+1}\, c_L \left[\left[\hat{d}^\dagger \times \hat{d}^\dagger\right]^{(L)} \times \left[\hat{\tilde{d}} \times \hat{\tilde{d}}\right]^{(L)}\right]_0^{(0)} + \frac{1}{2}\upsilon_0 \left[\left[\hat{d}^\dagger \times \hat{d}^\dagger\right]^{(0)} \times \left[\hat{\tilde{s}} \times \hat{\tilde{s}}\right]^{(0)} + \left[\hat{s}^\dagger \times \hat{s}^\dagger\right]^{(0)} \times \left[\hat{\tilde{d}} \times \hat{\tilde{d}}\right]^{(0)}\right]_0^{(0)} + \frac{1}{\sqrt{2}}\upsilon_2 \left[\left[\hat{d}^\dagger \times \hat{d}^\dagger\right]^{(2)} \times \left[\hat{\tilde{d}} \times \hat{\tilde{s}}\right]^{(2)} + \left[\hat{d}^\dagger \times \hat{s}^\dagger\right]^{(2)} \times \left[\hat{\tilde{d}} \times \hat{\tilde{d}}\right]^{(2)}\right]_0^{(0)} + \frac{1}{2}u_0 \left[\left[\hat{s}^\dagger \times \hat{s}^\dagger\right]^{(0)} \times \left[\hat{\tilde{s}} \times \hat{\tilde{s}}\right]^{(0)}\right]_0^{(0)} + u_2 \left[\left[\hat{d}^\dagger \times \hat{s}^\dagger\right]^{(2)} \times \left[\hat{\tilde{d}} \times \hat{\tilde{s}}\right]^{(2)}\right]_0^{(0)} \quad (1)$$

There are one-body of two terms definite by the parameters $\varepsilon_s, \varepsilon_d$ and seven terms of two –body, definite by the parameters $\upsilon_l(L = 0,2)$, $u_l(L = 0,2)$, c_L (L=0,2,4) to this other. There are three, and only three group reductions that can be constructed, is represented by the U (5) ,SU (3) and O(6) chains corresponding to three different illustrations of nuclear collective quadruple motion.[9]

1-Dynamical symmetry U(5)

There are 25 generators $[d^+ \times d]_\mu^{(L)}$close under commutation and form the sub algebra U (5) with number of generator (1/2n(n-1)) of U(6) . Furthermore, the ten components $[d^+ \times d]^{(1)}$ and $[d^+ \times d]_\mu^{(3)}$ close under commutation and the generators of

the vibrational limit O(5). Here, the $[d^{+}\times d]^{(1)}$ are the generators of O (3) and the$[d^{+}\times d]_{0}^{(1)}$the generators of O (2) algebra [10].

$$\begin{array}{ccccccc} U(6) & \supset U(5) & \supset O(5) & \supset O(3) & \supset O(2) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ [N] & n_d & v, n_\Delta & L & M_L & & \end{array} \quad (2)$$

Its eigenvalue depends on its own quantum numbers is[11,12]:

$$E|N, n_d, \nu, n_\Delta, L, M\rangle = \varepsilon n_d + \alpha\frac{1}{2}n_d(n_d-1) + \beta(n_d-\nu)(n_d+\nu+3) + \gamma[L(L+1)-6n_d] \quad (3)$$

and The Hamiltonian operator for this limit in the convenient format of the multipole expansion :

$$\hat{H}^{I} = \varepsilon\hat{n}_d + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \quad (4)$$

Where $\hat{n}_d$, $\hat{L}$, $\hat{T}_3$and $\hat{T}_4$are the total number of d-boson ,angular momentum , octoupole and hexadecapole operator respectively which defined by the following equations:

$\hat{n}_d = [\hat{d}^{\dagger}.\tilde{\hat{d}}]$, $\hat{L} = \sqrt{10}[\hat{d}^{\dagger}\times\tilde{\hat{d}}]^{(1)}$, $\hat{T}_3 = [\hat{d}^{\dagger}\times\tilde{\hat{d}}]^{(3)}$, $\hat{T}_4 = [\hat{d}^{\dagger}\times\tilde{\hat{d}}]^{(4)}$and $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy and the parameters a_1,a_3,a_4 relative to each limit of Hamilton [12] .

2-Dynamical symmetry O(6)

The 15 generators $[d^{+}\times d]_{\mu}^{(1)}$, $[d^{+}\times d]_{\mu}^{(3)}$, $[d^{+}\times s+ s^{+}\times d]_{\mu}^{(2)}$close under commutation and form the O (6) sub algebra.

And complement the series algebraic is like the series algebraic of vibrational limit

$$\begin{array}{ccccccc} U(6) & \supset O(6) & \supset O(5) & \supset O(3) & \supset O(2) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ [N] & \sigma & \tau, v_\Delta & L & M_L & & \end{array} \quad (5)$$

Its eigenvalue depends on its own quantum numbers is[13]**:**

$$E|N, \sigma, \tau, \nu_\Delta, L, M_L\rangle = A(N-\sigma)(N+\sigma+4) + B\tau(\tau+3) + CL(L+1) \quad (6)$$

and The Hamiltonian operator for this limit in the convenient format of the multipole expansion is :

$$\hat{H}^{III} = a_0\hat{p}.\hat{p} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \quad (7)$$

Where $\hat{p}$ represent the pairing operator and equal to($0.5[\hat{\tilde{d}}.\tilde{\hat{d}} - \hat{\tilde{s}}.\hat{\tilde{s}}]$) and a_0 is pairing parameter. The Hamiltonian operator for transition region between two dynamical symmetry U(5)-O(3) is:

$$\hat{H}^{I-III} = \varepsilon n_d + a_0\hat{p}.\hat{p} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \quad (8)$$

The ratio of (ε / a_0) determines the properties of this type. When this ratio is large, this means that the nucleus is close to its properties of U (5) and when it is small, it approaches the nucleus in its properties of determination O (6).

In addition to energy levels, the electromagnetic transfer rate can be calculated using this model and can be written in the following equation[14]:

$$\hat{T}_{\mu}^{(L)} = \alpha_2 \delta_{L2} \left[d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d} \right]_{\mu}^{(2)} + \beta_2 \left[d^{\dagger} \times \tilde{d} \right]_{\mu}^{(L)} + \gamma_{\circ} \delta_{L0} \delta_{\mu 0} [s^{\dagger} \times s]_0^{(0)} \qquad (9)$$

Where α_2, β_l, γ_0 are the coefficient of the various terms in the operator .This equation yields transition operators for E0,M1,E2,M3and E4 transition with appropriate value of the corresponding parameters .
The quadruple electric operator (E2) for each limit take the following formula[15]:

$$\hat{T}_{\mu}^{(E_2)} = \alpha_2 [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]_{\mu}^{(2)} + \beta_2 [d^{\dagger} \times \tilde{d}]_{\mu}^{(2)} \ldots\ldots\ldots\ldots\ldots U(5) \qquad (10)$$

$$T_{\mu}^{(E_2)} = \alpha_2 [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]_{\mu}^{(2)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots O(6)$$
(11)

The reduced value of the probability of quadruple electric transition B(E2) along the ground state band is:

$$B(E2; L_i \longrightarrow L_f) = \frac{1}{2L_i+1} \left| \langle L_f \| \hat{T}_{\mu}^{(E_2)} \| L_i \rangle \right| \qquad (12)$$

Potential Energy Surface

A general expression for the energy surface stated in term of the Hamiltonian of equation 4 and 7 is giving by :

$$E^{(I)}(N; \beta, \gamma) = E_0 + \varepsilon_d N \frac{\beta^2}{(1+\beta^2)} + f_1 N(N-1) \frac{\beta^4}{(1+\beta^2)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots U(5) \qquad (13)$$

$$E^{(III)}(N; \beta, \gamma) = E_0 + (2B + 6C) \frac{N\beta^2}{(1+\beta^2)} + \frac{A}{4} N(N-1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots O(5) \qquad (14)$$

Clear in U(5) limit the energy minimum for the ground state at β=0 and in O(6) limit is at β=1 [16].

Results and Discussion

The energy levels of even- even $^{68\text{-}70}$Zn isotopes have been calculated, by using the experimental energy ratios within the framework of IBM-1. It has been found that the ^{70}Zn isotope is not deformed nuclei and has a dynamical symmetry U(5), but the ^{68}Zn isotope is transitional nucleus between two limit U(5) and O(6) **.** In IBM-1, the even $^{68\text{-}70}$Zn isotopes have a number of proton boson particles equal to 1, and a number of neutron boson particle 5 and neutron boson hole 5 also ,respectively.

The parameters of equation (4) and (8) were calculated from the experimental schemes of these nuclei [17] and eigenvalue equations (6,3).These parameters were tabulated in table (1)

The Table (1) shows that the parameters used in the present work

Parameters / Isotope	N	ε	a_0	a_1	a_3	a_4
$^{68}_{30}Zn_{38}$	6	1.0773	0.4400	0.12563	0.7040	0
$^{70}_{30}Zn_{40}$	6	0.8867	0	0.0045	0.0973	-0.076

The calculated energy level and the experimental data on low-lying states are plotted in Figs (1) for even-even $^{68\text{-}70}$Zn isotopes. and The theoretical and experimental values are presented in Table 2 These figures show that the IBM-1 calculations of the energies, spin, and parity are in good agreement with the experimental values[17]. Levels with '( )' correspond to the cases, for which the spin and/or parity of the corresponding states are not well established experimentally.

table (2) comparison of experimental energy levels [17] and the IBM-1 calculations with correlation coefficient (R) of each one.

^{68}Zn (R=0.99)			^{70}Zn(R=0.995)		
J^P	IBM-1	exp.	J^P	IBM-1	exp.
0^+_1	.000	.000	0^+_1	.000	.000
2^+_1	1.073	1.077	2^+_1	0.886	0.884
0^+_2	1.665	1.655	0^+_2	1.17	1.07
2^+_2	2.084	1.883	2^+_2	1.823	1.759
2^+_3	2.73	2.338	4^+_1	1.826	1.786
4^+_1	2.23	2.417	2^+_3	1.95	1.957
3^+_1	3.29	3.009	0^+_3	2.126	2.14
0^+_3	3.16	3.102	4^+_2	2.682	2.693
4^+_2	3.38	3.281	6^+_1	2.82	(2.895)
6^+_1	3.624	(3.687)	3^+_1	2.856	(2.949)
5^+_1	4.73	4.345	6^+_2	5.518	(3.598)
6^+_2	4.952	4.963	8^+_1	3.8682	(3.7554)
-	-	-	10^+_1	4.969	4.935

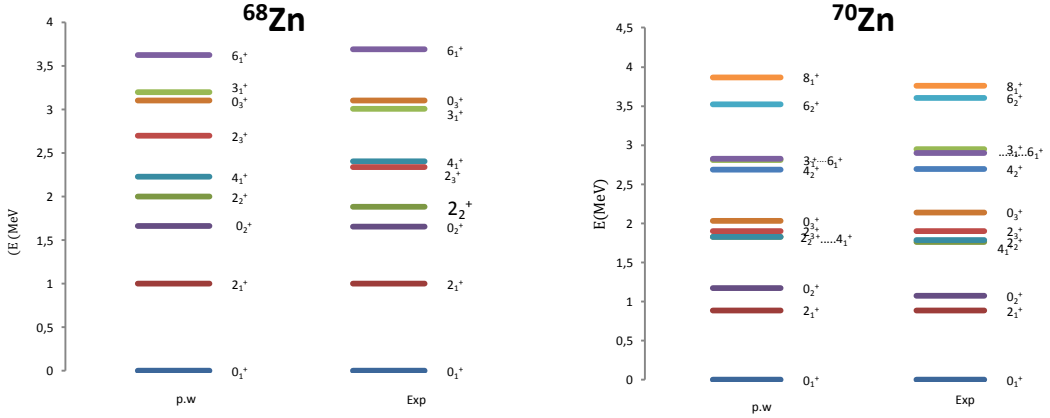

Figure (1) shows comparing the energy levels of the current study with the practical values [17] for the $^{68\text{-}70}$Zn isotope

The calculations of B(E2) values were performed using "IBMT- code". The parameters in (E2) operator eq.(10) and (11) were determined by fitting the experimental B(E2;21+□01+) data and the parameters were listed in table(3) where $\beta_2 = -\frac{0.7}{\sqrt{5}}\alpha_2, 0 \quad E2SD = \alpha_2, E2DD = \sqrt{5}\beta_2$ .The values of the parameters which gave the best fit to experimental[18] are given in table (4).

Table (3) The parameters of the Hamiltonian equation and B(E2;$2_1^+ \rightarrow 0_1^+$) for $^{68\text{-}70}$Zn isotopes

Nucleus	B(E2;$2_1^+ \rightarrow 0_1^+$)e^{2}b^2	E2SD	E2DD
$^{68}_{30}Zn_{38}$	0.0241	0.04197	-0.05568
$^{70}_{30}Zn_{40}$	0.0282	0.04527	-0.10668

Table (4) Comparison between present values of B(E2) for even-even $^{68\text{-}70}$Zn isotopes (p.w) and experimental ones (Exp.)

^{68}Zn	B(E2) e^{2}b^2		^{70}Zn	B(E2) e^{2}b^2	
$J_i \rightarrow J_f$	Exp	p.w	$J_i \rightarrow J_f$	Exp	p.w
$2_1^+ \rightarrow 0_1^+$	0.0240	0.0246	$2_1^+ \rightarrow 0_1^+$	0.0282	0.0282
$2_1^+ \rightarrow 0_2^+$	0.0018	0.0011	$2_1^+ \rightarrow 0_2^+$	0.0018	0.0007
$2_1^+ \rightarrow 0_3^+$		0.0016	$2_1^+ \rightarrow 0_3^+$		0.0028
$2_2^+ \rightarrow 0_1^+$	0.0013	0.0012	$2_2^+ \rightarrow 0_1^+$	0.003	0.0039
$2_2^+ \rightarrow 0_2^+$		0.0002	$2_2^+ \rightarrow 0_2^+$		0.0003
$2_2^+ \rightarrow 0_3^+$	--------	0.0107	$2_2^+ \rightarrow 0_3^+$	--------	0.0181
$2_3^+ \rightarrow 0_2^+$	0.0055	0.0128	$2_3^+ \rightarrow 0_2^+$		0.0121
$2_3^+ \rightarrow 0_3^+$	--------	0.0005	$2_3^+ \rightarrow 0_3^+$	--------	0.0017
$2_4^+ \rightarrow 0_1^+$	0.00009	0.0000	$2_4^+ \rightarrow 0_1^+$	0.0002	0.0000
$2_4^+ \rightarrow 0_2^+$	0.0002	0.0001	$2_4^+ \rightarrow 0_2^+$		0.0003
$2_4^+ \rightarrow 0_3^+$	--------	0.02	$2_4^+ \rightarrow 0_3^+$	--------	0.0271
$2_1^+ \rightarrow 2_2^+$	--------	0.0380	$2_1^+ \rightarrow 2_2^+$	--------	0.0454
$2_1^+ \rightarrow 2_3^+$	0.0001	0.0002	$2_1^+ \rightarrow 2_3^+$		0.0006
$2_2^+ \rightarrow 2_1^+$	0.0439	0.0380	$2_2^+ \rightarrow 2_1^+$	0.0236	0.0454
$2_2^+ \rightarrow 2_3^+$	--------	0.0007	$2_2^+ \rightarrow 2_3^+$	--------	0.0002
$2_3^+ \rightarrow 2_1^+$	0.0001	0.0002	$2_3^+ \rightarrow 2_1^+$		0.0006
$2_3^+ \rightarrow 2_2^+$	--------	0.0007	$2_3^+ \rightarrow 2_2^+$	--------	0.0002
$2_4^+ \rightarrow 2_1^+$	0.0001	0.0001	$2_4^+ \rightarrow 2_1^+$	0.0047	0.0002
$2_4^+ \rightarrow 2_2^+$	0.0031	0.0030	$2_4^+ \rightarrow 2_2^+$		0.0062
$2_4^+ \rightarrow 2_3^+$		0.0004	$2_4^+ \rightarrow 2_3^+$		0.0017
$4_1^+ \rightarrow 2_1^+$	0.0196	0.0359	$4_1^+ \rightarrow 2_1^+$	0.0407	0.0408
$4_1^+ \rightarrow 2_2^+$	0.009	0.0011	$4_1^+ \rightarrow 2_2^+$		0.0042
$4_1^+ \rightarrow 2_3^+$		0.0013	$4_1^+ \rightarrow 2_3^+$		0.0010
$4_2^+ \rightarrow 2_1^+$		0.0009	$4_2^+ \rightarrow 2_1^+$	0.0043	0.0029
$4_2^+ \rightarrow 2_2^+$	--------	0.0226	$4_2^+ \rightarrow 2_2^+$	0.0333	0.0266
$2_3^+ \rightarrow 0_1^+$	0.00003	0.0000	$4_2^+ \rightarrow 2_3^+$		0.0015
$4_3^+ \rightarrow 2_2^+$		0.0010	$4_3^+ \rightarrow 2_2^+$		0.0033
$4_3^+ \rightarrow 2_3^+$		0.0001	$4_3^+ \rightarrow 2_3^+$		0.0004
$Q2_1^+$	-0.106	-0.1067	$Q2_1^+$	-0.233	-0.2338

The potential energy surface gives a final shape to the nucleus that corresponds to the Hamiltonian in the equations (13), (14) The values in Table (5) show coefficients used in IBMP-code to calculate surface energy potential E(N, β,γ) also draw the energy functional E(N; β,γ)as a function of β and the contour plots in the γ-β plane in the fig.(3)

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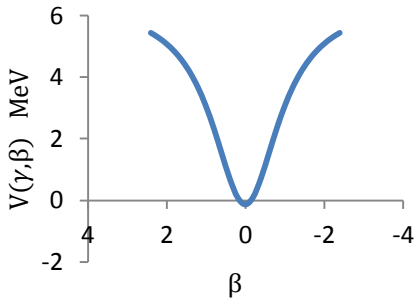


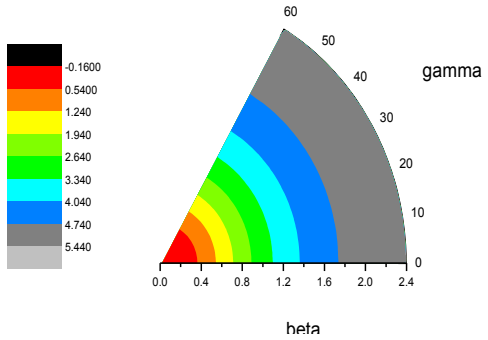
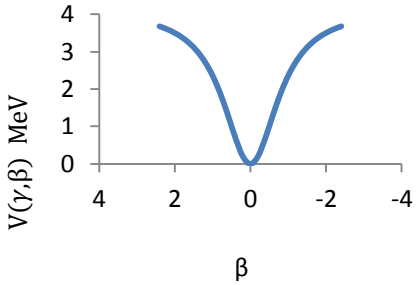


Fig.(3):The energy functional E(N; β,γ) as a function of β and the corresponding β-γ plot for ^{68}Zn isotopes.



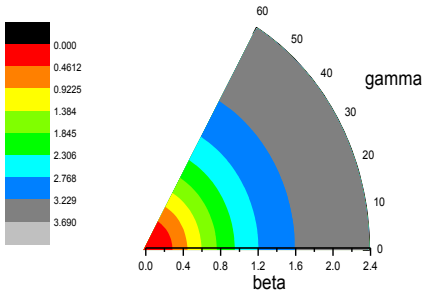


Fig.(3):The energy functional E(N; β,γ) as a function of β and the corresponding β-γ plot for ^{70}Zn isotopes.

Conclusions

The result of current study confirmed that the nuclear ^{68}Zn have showed overlapping behavior between the vibrational characteristics and collective characteristics there considered as transitional nucleus between two limit U(5) and O(6) and defer from ^{70}Zn nucleus which has mostly vibrational behavior .In addition the result of couture line and axillary symmetries show the there is no nuclear deformation and prove that the nucleus that have been studied are transition nuclear between spherical shape and gamma instable defer from ^{70}Zn nucleus which has spherical shape only.

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