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STABILITY OF STEADY STATE MOTION OF VIBROPROTECTED PLATE WITH HYSTERESIS CHARACTERISTICS

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Abstract: In this article, we will explore absolute stability of steady state solution of nonlinear motion of 45 mark steel plate with elastic dynamic absorber, which is put on the geometric centre of the plate in harmonic excitations. Namely, we will investigate the condition and the border of stability according to changeable parameters of the plate and the dynamic absorber.
Key words: Nonlinear plate; Dynamic absorber; Vibration; Steady state solution; Absolute stability.
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Introduction: In general, stability problems are very important practically to predict the future motion of moving system. Theoretical bases of dynamics and

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nonlinear motion stability are expressed in many sources [1-7]. In monograph [1], it is analysed dynamics of nonlinear plate and dynamic absorber. Differential equations of motion and their solutions are received and analysed at various changeable parameters of system. In the book [2] normal equations, steady state solution, it's stability and Liapunov's stability methods are showed. The book [3] gives us information about stability theory and some solved problems in nonlinear mechanical system. The methodology how to get the border and condition of stability is showed for a single degree of freedom and multi degrees of freedom systems. The theory of anisotropic plate is in the book [4]. General notions about anisotropic plate,it's free vibration and stability of free vibration of anisotropic plate are given in it. More information are given about dynamic absorber in the book [5]. We can find information about any type dynamic absorber and it's properties.

We will discuss stability of steady state solution of motion of 45 mark steel plate with dynamic absorber in Fig.1.

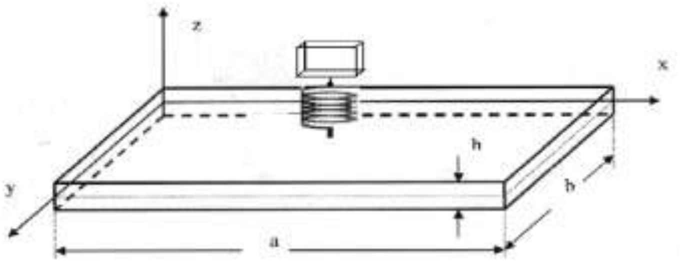

Fig.1. Plate and dynamic absorber

Tab.1.Geometric sizes and mechanical properties of plate and dynamic absorber

a/m	b/m	h/m	$\rho/kg\cdot m^{-3}$	$E/N\,m^{-2}$	$G/N\,m^{-2}$	μ /dimensionless	M/kg	$c/N\,m^{-1}$
0.19	0.19	0.001	7810	$2.08\cdot10^{11}$	$0.8\cdot10^{11}$	0.3	0.5	10^6

Results and Discussions: We use the differential equations of motion of hysteretic plate with hysteretic type dynamic absorber [1]. It will be as following when $u_{ik}(x,y)=u_{11}(x,y)$ for our problem:

$$\begin{cases}\ddot{x}_{11}+\left(1+T_1(-\eta_1+j\eta_2)+T_2(-\nu_1+j\nu_2)\right)p_{11}^2x_{11}-d_{3,11}x_2=-d_{11}W_0;\\ \ddot{x}_{11}+\ddot{x}_2+n^2x_2=-W_{0,}\end{cases}\quad(1)$$

where

x_{11} =displacement of $x_{ik}(i=k=1)$ point of plate (m)

$T_1 = 31.43047x_{11a}+3003.318x_{11a}^2-6935521x_{11a}^3$ (*dimensionless*)

$T_2 = -0.133987x_{11a}^2$ (*dimensionless*)

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$x_{11a} = x_{11a}(t)$ =amplitude of x_{11} (m)
$j^2 \;\; = -1$
$\eta_1 \;\; = \;\; 142.717x_{11a} + 22660477x_{11a}^2 - 1.74149 \cdot 10^{12}x_{11a}^3$ $(dimensionless)$
$\eta_2 = 60.57097x_{11a} + 9617405x_{11a}^2 - 2.7389243 \cdot 10^{12}x_{11a}^3(dimensionless)$
$\nu_1 \;\; = \;\; 0.613245x_{11a} - 62.7862x_{11a}^2 + 4240.29x_{11a}^3$ $(dimensionless)$
$\nu_2 \;\; = \;\; 0.26027x_{11a} - 26.6473x_{11a}^2 + 1799.635x_{11a}^3$ $(dimensionless)$
$p_{11} = 847.48$ =natural frequency of $x_{ik}(i = k = 1)$ point of plate (s^1)
$u_{11}(x,y) = sin\frac{\pi x}{0.19} sin\frac{\pi y}{0.19}$ = natural vibration form of $x_{ik}(i = k = 1)$ point of plate $(dimensionless)$

$$d_{1,11} = \iint u_{11}dxdy = \frac{4 \cdot 0.19^2}{\pi^2}(m^2)$$

$$d_{2,11} = \iint u_{11}^2 dxdy = \frac{0.19^2}{4} \;(m^2)$$

$$d_{3,11} = \frac{c}{\rho h d_{2,11}} = 14187365.44 \;(s^{-2})$$

$$d_{11} = \frac{d_{1,11}}{d_{2.11}} = \frac{16}{\pi^2} \;(dimensionless)$$

$n = \sqrt{\frac{c}{M}}$ =1414.214= natural frequency of dynamic absorber (s^{-1})
$W_0 = \xi cos\omega t$= acceleration of foundation (ms^{-2})
ξ=0.001=maximum quantity of W_0 (ms^{-2})
ω = frequency of system (s^{-1})

We will get solution of (1) as following:

$$\begin{cases} x_{11} = x_{11a}\cos(\omega t + \alpha_{11})\,; \\ x_2 = x_{2a}\cos(\omega t + \alpha_2)\,, \end{cases} \tag{2}$$

where
$\alpha_{11} = \alpha_{11}(t)$ =phase of $x_{ik}(i = k = 1)$ point of plate $(dimensionless)$

$x_{2a} = x_{2a}(t)$ = amplitude of x_2 (m)

$\alpha_2 = \alpha_2(t)$ = phase of x_2 $(dimensionless)$.

$x_{11a}(t)$,$\alpha_{11}(t)$, $x_{2a}(t)$, $\alpha_2(t)$ are counted functions [2] that are changing slowly. According to character of these functions, it is possible to write following equalities:

$$\ddot{x}_{11a} = \ddot{\alpha}_{11} = \ddot{x}_{2a} = \ddot{\alpha}_2 = \dot{x}_{11a}\dot{\alpha}_{11} = \dot{x}_{2a}\dot{\alpha}_2 = \dot{x}_{11a}\dot{x}_{2a} = \dot{x}_{11a}\dot{\alpha}_2 = \dot{\alpha}_{11}\dot{x}_{2a} == \dot{\alpha}_{11}\dot{\alpha}_2 = 0 \tag{3}$$

We will put (2) into (1) according to (3). Then we can get normal equations of (1).

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$$\begin{cases} \dot{x}_{11a} = \frac{1}{2\omega}\left(d_{11}\xi sin\alpha_{11} - p_{11}^2 x_{11a}(T_1\eta_2 + T_2\nu_2) + d_{3,11}x_{2a}\sin(\alpha_2 - \alpha_{11})\right); \\ \dot{\alpha}_{11} = \frac{1}{2\omega x_{11a}}(d_{11}\xi cos\alpha_{11} + p_{11}^2 x_{11a}(1 - T_1\eta_1 - T_2\nu_1) - x_{11a}\omega^2 - \\ \qquad -d_{3,11}x_{2a}\cos(\alpha_2 - \alpha_{11})); \\ \dot{x}_{2a} = \frac{1}{2\omega}((1 - d_{11})\xi sin\alpha_2 - p_{11}^2 x_{11a}(1 - T_1\eta_1 - T_2\nu_1)\sin(\alpha_2 - \alpha_{11}) + \\ \qquad +p_{11}^2 x_{11a}(T_1\eta_2 + T_2\nu_2)\cos(\alpha_2 - \alpha_{11})); \\ \dot{\alpha}_2 = \frac{1}{2\omega x_{11a}}((1 - d_{11})\xi cos\alpha_2 + x_{2a}(n^2 + d_{3,11} - \omega^2) - p_{11}^2 x_{11a} \times \\ \times \left((T_1\eta_2 + T_2\nu_2)\sin(\alpha_2 - \alpha_{11}) + (1 - T_1\eta_1 - T_2\nu_1)\cos(\alpha_2 - \alpha_{11})\right)). \end{cases} \quad (4)$$

Steady state solution of (1) will be found from (4).

$$\begin{cases} x_{11a} = \xi \frac{\left|n^2 d_{11} + d_{3,11} - \omega^2 d_{11}\right|}{\sqrt{\Delta}}; \\ x_{2a} = \xi \frac{\sqrt{(p_{11}^2(1 - T_1\eta_1 - T_2\nu_1) - (1 - d_{11})\omega^2)^2 + (p_{11}^2(T_1\eta_2 + T_2\nu_2))^2}}{\sqrt{\Delta}}, \end{cases} \quad (5)$$

where

$$\Delta = (\omega^4 - \left(n^2 + d_{3,11}\right)\omega^2 + (1 - T_1\eta_1 - T_2\nu_1)p_{11}^2(n^2 - \omega^2))^2 + (p_{11}^2(T_1\eta_2 + T_2\nu_2)(n^2 - \omega^2))^2.$$

As it is stated above, we will explore stability of steady state solution by means of Liapunov's method [2,6,7]. According to this method, we should calculate the variation of (4) and then we can get following characteristic equation:

$$(2\omega\lambda)^4 + b_0(2\omega\lambda)^3 + b_1(2\omega\lambda)^2 + b_2(2\omega\lambda)^1 + b_3(2\omega\lambda)^0 = 0, \quad (6)$$

where

λ = characteristic number

$$b_0 = p_{11}^2(T_1\eta_2 + T_2\nu_2 + \frac{\partial}{\partial x_{11a}}(x_{11a}(T_1\eta_2 + T_2\nu_2)))$$

$$b_1 = \left((1 - T_1\eta_1 - T_2\nu_1)\frac{\partial}{\partial x_{11a}}\left(x_{11a}(1 - T_1\eta_1 - T_2\nu_1)\right) + (T_1\eta_2 + T_2\nu_2)\frac{\partial}{\partial x_{11a}}\left(x_{11a}(T_1\eta_2 + T_2\nu_2)\right)\right)p_{11}^4 + (1 - T_1\eta_1 - T_2\nu_1 + \frac{\partial}{\partial x_{11a}}\left(x_{11a}(1 - T_1\eta_1 - T_2\nu_1)\right))\left(d_{3,11} - \omega^2\right)p_{11}^2 + \left(n^2 + d_{3,11} - \omega^2\right)^2 + \omega^4$$

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$$b_2 = b_0((n^2-\omega^2)^2 + n^2 d_{3,11})$$

$$b_3 = \left((1 - T_1\eta_1 - T_2\nu_1)\frac{\partial}{\partial x_{11a}}\big(x_{11a}(1 - T_1\eta_1 - T_2\nu_1)\big) + (T_1\eta_2 + T_2\nu_2)\frac{\partial}{\partial x_{11a}}\big(x_{11a}(T_1\eta_2 + T_2\nu_2)\big)\right)(n^2-\omega^2)^2 p_{11}^4 - (1 - T_1\eta_1 - T_2\nu_1 + \frac{\partial}{\partial x_{11a}}\big(x_{11a}(1 - T_1\eta_1 - T_2\nu_1)\big))(n^2 - \omega^2)(\omega^2 - n^2 - d_{3,11})p_{11}^2\omega^2 + \left(\omega^2 - n^2 - d_{3,11}\right)^2\omega^4$$

According to Liapunov's theorem [2], steady state solution of motion is asymptotic stable when the real part of squares of (6) must be positive sign. Hurwitz's theorem [3]can answer to this question. Hurwitz's criterion will be as following for (6):

$$b_0 > 0, b_1 > 0, b_2 > 0, b_3 > 0, b_0 b_1 b_2 - b_0^2 b_3 - b_2^2 > 0 \qquad (7)$$

It is possible that to say b_0 is always positive for all $x_{11a} \ll 1$ because it is depend on losing the energy of material. We can see it by it's graphic Fig.2 and b_2 is also positive as $b_0 > 0$.

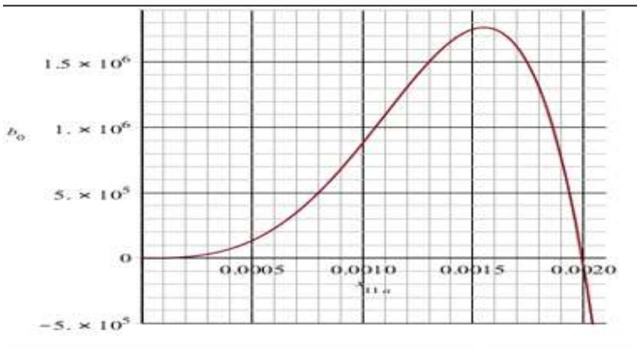

Fig.2. Graphic of the function b_0

We can get following condition that is general for other left three inequalities to be positive:

$$(x_{11a}\frac{\partial}{\partial x_{11a}}(T_1\eta_1 + T_2\nu_1))^2 - 4(T_1\eta_2 + T_2\nu_2)\frac{\partial}{\partial x_{11a}}\big(x_{11a}(T_1\eta_2 + T_2\nu_2)\big) < 0 \qquad (8)$$

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Received (8) is condition of stability. We can see two types intervals from it's graphic Fig.3. The one is the interval of stable amplitude and the second one is the interval of unstable amplitude.

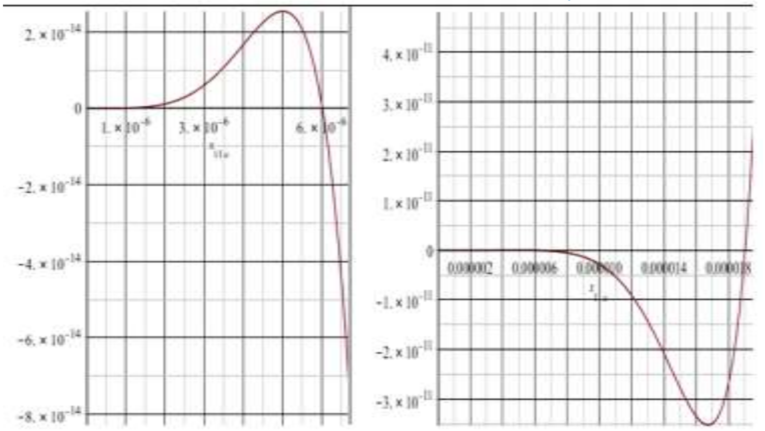

Fig.3. Graphic of condition of stability

We can get border of stability of given system from (7).

$$\left(\omega^4-\left(n^2+d_{3,11}\right)\omega^2\right)^2+\left(1-T_1\eta_1-T_2\nu_1+\frac{\partial}{\partial x_{11a}}\left(x_{11a}\left(1-T_1\eta_1-T_2\nu_1\right)\right)\right)\left(\omega^4-\left(n^2+d_{3,11}\right)\omega^2\right)p_{11}^2\left(n^2-\omega^2\right)+\left(\left(1-T_1\eta_1-T_2\nu_1\right)^2+\left(T_1\eta_2+T_2\nu_2\right)^2+\frac{x_{11a}}{2}\frac{\partial}{\partial x_{11a}}\left(\left(1-T_1\eta_1-T_2\nu_1\right)^2+\left(T_1\eta_2+T_2\nu_2\right)^2\right)\right)\left(p_{11}^2\left(n^2-\omega^2\right)\right)^2=0 \qquad (9)$$

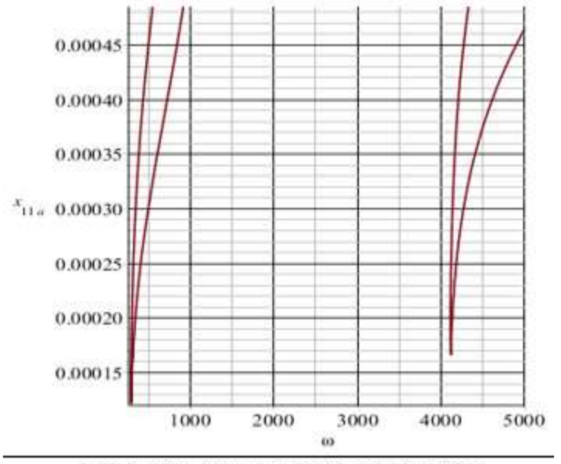

Fig.4. Graphic of border of stability

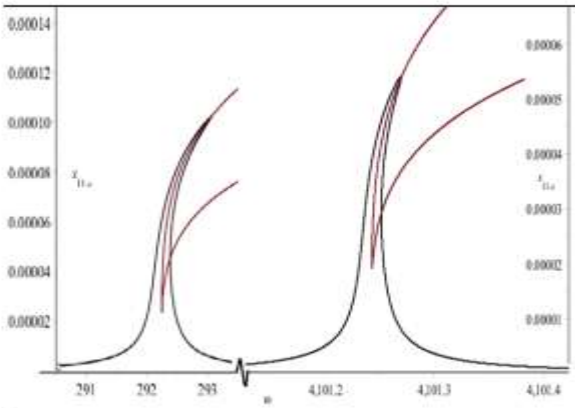

Fig.5. Vibration form and border of stability

We can see the border of stable and unstable motion from Fig.4. Between two lines are counted unstable region and the lines are counted border. It is very important practically that to see dynamical behavior of system. It is possible that to show the graphic of vibration form (5) and border of stability (9) by Fig.5.

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Conclusion: Steady state solution of given dynamic system has stable and unstable amplitudes. It is known us from received results that the steady state solution is stable for all amplitudes which are in $6 \cdot 10^{-6} \leq x_{11a} \leq 19 \cdot 10^{-6}$and unstable for all other amplitudes. Stable region exists in$2 \cdot 10^{-5} \geq x_{11a}$, $292 \geq \omega$, $293 \leq \omega \leq 4101and4102 \leq \omega$. Unstable region exists in$2 \cdot 10^{-5} < x_{11a}$, $292 < \omega < 293$ and $4101 < \omega < 4102$.

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