

PULSATING FLOWS OF A VISCOUS FLUID IN A FLAT CHANNEL WITH VARIABLE WALLS

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Abstract: In the article, pulsating fluid flows in tubes with changing walls, which are of great importance in biological mechanics, in particular, in the flow of blood in the arterial vessel, are investigated. The solution of the problem obtained formulas for the distribution of pressure gradient, velocity and flow rate. Impedance method analyzed the increase in hydraulic resistance depending on the frequency of oscillation. It was found that with large values α^2 of hydraulic resistance grow with growth α^2 . For crowding out the same flow rates, for small and with large values of the vibrational number or frequency, the energy input is different. Since for small values of the oscillatory number, fluctuations in the flow rate of the fluid fluctuate in the same phase with the oscillation of the pressure gradient, and for large values of this number, the oscillation of the phase of the fluid flow is shifted by 90° degrees than fluctuations in the pressure gradient. In addition, for large values of the vibrational number, a tenfold size increases the maximum amplitude of the pressure gradient, relatively, than at low frequencies. This feature leads to an increase in the energy cost of pumping fluid due to wall oscillation.

Keywords: Pipe, fluid, flow, biomechanics, arterial, blood vessels, blood, pressure, gradient, fluid flow, Impedance, hydraulic resistance, power consumption.

Introduction

During the contraction of the heart, a pressure wave propagates through the walls of the blood vessels, which is called the pulse wave. This wave gradually weakens with distance from the heart and practically dies out in the capillaries. The speed of propagation of a pulse wave depends on many factors, among which we can note, for example, the elastic-viscous properties of the vessel wall, blood pressure, its density, viscosity, wall permeability, active modified vessels. In the majority of works [1-6], the main attention is paid to the determination of the propagation of a pressure pulse wave taking into account the elasticity of the vessel wall, and its active change has never been taken into account. However, the active change in the wall significantly affects the propagation of the pressure pulse wave and its attenuation. In [1-6], problems for pulsating fluid flow in tubes with varying walls with a low frequency are solved. In works [11-14], the impedance method analyzed the decrease in

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hydraulic resistance in pipes with permeable walls. This article solves the problem in a generalized form, where the oscillatory parameter can be significant.

Research Methodology

Using those techniques, as in [7–9], after some transformations of the quantities, the Navier – Stokes system of equations and the continuity equations are written in dimensionless form, which contain, with boundary conditions, two dimensionless quantities. Among these values, the relative amplitude (the ratio of the maximum deflection of wall δ_m to the width of pipe h_0 , $\ell = \frac{\delta_m}{h_0}$) was chosen as the main parameter; the remaining values are expressed in terms of the positive degrees of the main small parameter using ratios

$$\varepsilon = \varepsilon_0 \ell^n, \quad n \geq 1. \quad (1)$$

Below, using this method, we solve the Navier-Stokes equations and the continuity equations by the perturbation method. We search for unknown longitudinal and transverse velocities and pressures in the form of expansion in powers of the main small parameter ℓ

$$u = \sum_{m=0}^{\infty} u_m \ell^m, \quad v = \sum_{m=0}^{\infty} v_m \ell^m, \quad p = \sum_{m=0}^{\infty} p_m \ell^m. \quad (2)$$

Substituting (1) in view of equality (2) into the system of Navier-Stokes equations and the equations of continuity [1], and comparing the same degrees of a small parameter, we obtain a sequence of systems. The resulting system of equations differs from [7-10], so that, there is saved a member containing parameter α^2 . In addition to the zero approximation, the other equations depend on a small parameter. Therefore, we can express the following approximations, through the previous ones. To determine u_i, v_i, p_i in all approximations we write

$$\begin{aligned} \frac{\partial^2 u_i}{\partial r^2} + \frac{1}{r} \frac{\partial u_i}{\partial r} &= \frac{\partial p_i}{\partial x} + \alpha^2 \frac{\partial u_i}{\partial t} + \varphi_i(x, y, t), \\ \frac{\partial p_i}{\partial r} &= \psi(x, y, t), \quad \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial r} + \frac{v_i}{r} = 0 \end{aligned} \quad (3)$$

Where $\varphi_i(x,y,t)$ and $\psi_i(x,y,t)$ are determined using the previous approximations. All accepted designations here correspond to the notation [1]. The boundary conditions take the following form

$$\begin{aligned} \frac{\partial u_i}{\partial r} = 0, \quad v_i = 0 \quad \text{at } r = 0 \\ u_i = u_{wi}(x,y,t), \quad v_i = v_{wi}(x,y,t) \quad \text{at } r = 1 \end{aligned} \quad (4)$$

Here $u_{wi}(x,y,t)$, $v_{wi}(x,y,t)$ is located by decomposing unknown functions u_i , v_i into a Taylor series in a neighborhood of $r = 1$. Since the perturbation parameter ℓ is explicitly and implicitly included in the boundary conditions, it is impossible here to directly equate to zero terms with different powers of ℓ . Therefore, one can preliminarily decompose them into Taylor series in order to obtain their explicit dependence on ℓ . If we assume that functions u_{wi} , v_{wi} are analytic with respect to $r = 1$, then they can be expanded into Taylor series near $r = 1$

$$\begin{aligned} u_{wi}(x,1+\ell f,t) = u_{w0}(x,1,t) + \left(u_{w1} + f \left(\frac{\partial u_{w0}}{\partial r} \right)_{r=1} \right) \ell + \dots = 0 \\ v_{wi}(x,1+\ell f,t) = v_{w0}(x,1,t) + \left(v_{w1} + f \left(\frac{\partial v_{w0}}{\partial y} \right)_{y=1} \right) \ell + \dots = \frac{\partial f}{\partial t} \end{aligned} \quad (5)$$

For the zero, first, and second approximations (5) should be assumed.

$$\begin{aligned} u_{w0} = 0, \quad u_{w1} = -f \left(\frac{\partial u_{w0}}{\partial y} \right)_{y=1}, \quad u_{w2} = -f \left(\frac{\partial u_{w1}}{\partial y} \right)_{y=1} - \frac{f^2}{2} \left(\frac{\partial^2 u_{w0}}{\partial y^2} \right)_{y=1}, \\ v_{w0} = 0, \quad v_{w1} = -f \left(\frac{\partial v_{w0}}{\partial y} \right)_{y=1} + \frac{\partial f}{\partial t}, \quad v_{w2} = -f \left(\frac{\partial v_{w1}}{\partial y} \right)_{y=1} - \frac{f^2}{2} \left(\frac{\partial^2 v_{w0}}{\partial y^2} \right)_{y=1}. \end{aligned} \quad (6)$$

Above it was said that the flow of fluid is carried out only due to periodic oscillations of the wall, therefore, $\frac{\partial p_0}{\partial x} = 0$ and, therefore, $u_0 = 0$, $v_0 = 0$, and the remaining quantities are determined after simple calculations from the system of equations (3) and boundary conditions (6).

$$u_1 = \frac{1}{i\alpha^2} \left(-\frac{\partial p_1}{\partial x} \right) \left[1 - \frac{ch\sqrt{i}\alpha y}{ch\sqrt{i}\alpha} \right],$$

Analys and Results

Integrating the resulting formula (7) from 0 to 1, the longitudinal velocity u_1 , u_2 , we find the flow rate of the flowing fluid over sections x

$$Q = \ell Q_1 + \ell^2 Q_2; \quad \ell = \frac{\delta_m}{h_0}, Q = -\frac{\partial f}{\partial t} x, \quad Q_2 = 0. \quad (8)$$

Thus, here only Q_1 is an expense non-zero, the remaining expenses are equal to zero. Therefore, the total volumetric flow rate is determined from the formula.

$$Q = -\delta_m \frac{\partial f}{\partial t} x. \quad (9)$$

The results obtained in this case also prove the assertion that the costs inserted by moving the pipe walls are equal to the sum of the longitudinal costs flowing through the sections $x = -L$, $x = L$.

Indeed, the displaced transverse costs are determined by integrating the transverse velocities on the walls from $-L$ to L along the longitudinal coordinates

$$Q_{transvers.} = \int_{-L}^L [\delta_m v_1 + \delta_m^2 v_2] dx = 2\delta_m \frac{\partial f}{\partial t} L, \quad (10)$$

and the amount of longitudinal costs will be equal to the amount of expenses in sections $x = L$, $x = -L$ in absolute values

$$Q_{longitudinal.} = |Q_{x=-L}| + |Q_{x=L}| = 2\delta_m \frac{\partial f}{\partial t} L. \quad (11)$$

from formulas (10) and (11) it is clear that

$$Q_{longitudinal.} = Q_{transvers.} \quad (12)$$

Now we define the relationship of the pressure gradient to the fluid flow

$$\frac{(-\frac{\partial p}{\partial x})}{Q} = \frac{i\alpha^2}{2(1 - \frac{1}{\sqrt{i}\alpha})} = R + Li \quad (13)$$

The real part (13) determines the hydraulic resistance of the flow imaginary part of the attenuation of the wave. At lower values of α^2 hydraulic resistance is determined by the following formula.

$$R = \frac{3\mu}{2h^3}, \quad L = 0 \quad (14)$$

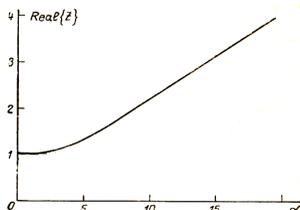


Fig. 1.

The dependence of the hydraulic resistance of the flow $Real\{Z\}$ from the parameter α .

Conclusion

From Fig.1. it is seen that for large values of α^2 hydraulic resistance increase with an increase of α^2 . For crowding out the same flow rates, for small and with large values of the vibrational number or frequency, the energy input is different. Since for small values of the vibrational number, fluctuations in the flow rate of the fluid fluctuate in the same phase with the oscillation of the pressure gradient, and for large values of this number, the fluctuations of the flow rate of the fluid flow are shifted 90° degrees than fluctuations in the pressure gradient [11-14]. In addition, for large values of the vibrational number, a tenfold size increases the maximum amplitude of the pressure gradient, relatively, than at low frequencies. This feature leads to an increase in the energy cost of pumping fluid due to wall oscillation.

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