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DETERMINATION OF AREAS OF DYNAMIC INSTABILITY OF INHOMOGENEOUS RODS.

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Abstract: The article uses transverse vibrations of a non-uniform rod on an elastic foundation as a result of the influence of periodically applied longitudinal forces. The problem was solved using the Bubnov-Galerkin method.

Keywords: Bubnov-Galerkin, OU, OZ,

The paper considers the problem of transverse vibrations of a straight rod located on a solid elastic foundation under the influence of periodic longitudinal forces. The problem was solved by the Bubnov-Galerkin method and the boundaries of the instability regions of inhomogeneous rods were determined.

The coordinate system is chosen as follows: the OX axis coincides with the line of action of the compressive load, the OU and OZ axes are located in the cross section of the rod.

Let us assume that the rod is made of an elastic, continuously inhomogeneous material, and the elastic modulus is a continuous function of the coordinates of length and thickness and varies according to the following law:

$$E = E_0 f_1(x) f_2(z) \qquad (1)$$

It is known that the equation of motion of the rod in question has the form

$$\Delta P = b \int_{-h/2}^{h/2} \Delta\sigma dz, \Delta M = b \int_{-h/2}^{h/2} \Delta\sigma z dz \qquad (2)$$

where m is the mass per unit length of the rod, is the deflection of the rod axis, is the base pushback coefficient (pull coefficient), the increments of force and moment are determined from the relations

$$\Delta P = b \int_{-h/2}^{h/2} \Delta\sigma dz, \Delta M = b \int_{-h/2}^{h/2} \Delta\sigma z dz \qquad (3)$$

here in width, h – height of the cross section of the rod.

Taking into account (1), the relationship between stress and strain increments is written as:

$$\Delta\sigma = E_0 f_1(x) f_2(z) \Delta\varepsilon \qquad (4)$$

We use the hypothesis of plane sections

$$\Delta\varepsilon = \varepsilon_0 + z\chi \qquad (5)$$

where is the additional extension of the rod axis, is the curvature of the center line.

Taking into account (4) and (5), from (3) we obtain:

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$$\Delta P = b(a^0\varepsilon_0 + a^1\chi) f_1(x); \Delta \mathrm{P} = b(a^1\varepsilon_0 + a^2\chi) f_1(x) \qquad (6)$$

The following notations are introduced in these formulas:

$$a^1 = \int_{-h|2}^{h|2} f_2(z) z^1 dz \quad , \quad (i = 0{,}1{,}2) \qquad (7)$$

Taking into account (6) from (1) the equation of system (2) we have:

Substituting this expression into (6) for the torque increment we obtain:

$$\Delta M = KIf_2(x)\chi \qquad (8)$$

where indicated

$$\Delta M = KIf_2(x)\chi \qquad (9)$$

Considering that , $\chi = \frac{\delta^2\sigma}{\delta x^2}$ the second equation of system (2) can be represented as:

$$KI\frac{\delta^2}{\delta x^2}\left[f_1(x)\frac{\delta^2\upsilon}{dx^2}\right] + P\frac{\delta^2\upsilon}{dx^2} + \beta\upsilon + m\frac{\delta^2\upsilon}{\delta t^2} = 0 \qquad (10)$$

As can be seen from (10), the motion equation is a fourth-order partial differential equation with variable coefficients. To solve the resulting equation, it is necessary to know the specific types of coefficients of this equation. Let's assume $f_1(x)$ that the function varies according to the law:

$$f_1(x) = 1 + \alpha\frac{x}{\lambda} \qquad (11)$$

In this case, the equation of motion (10) is transformed to the form:

$$KI(1 + \alpha\frac{x}{\lambda})\frac{\delta^4\upsilon}{\delta x^4} + 2KI\frac{\alpha}{\lambda}\frac{\delta^3\upsilon}{\delta x^3} + P\frac{\delta^2\upsilon}{\delta x^2} + \beta\upsilon + m\frac{\delta^2\upsilon}{\delta t^2} = 0 \qquad (12)$$

Let us consider a rod with hinged ends, in this case the boundary conditions have the form:

$$(\upsilon(0) = \upsilon(t) = 0; \qquad \upsilon^n(0) = \upsilon^n(\lambda) = 0 \qquad 13)$$

We look for the solution to equation (12) in the form:

$$\upsilon(x_1 t) = \varphi_n(t)\sin\frac{n\pi x}{\lambda} \qquad (14)$$

which satisfy the boundary conditions (13)

Substituting (14) into (12) and performing the procedure of the Bubnov – Galerkin method, we obtain:

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$$KI(1+\frac{\alpha}{2})(\frac{n\pi}{\lambda})^4\varphi_n(t)+m\frac{d^2\varphi_n}{dt^2}-\mathrm{P}\varphi_n(t)(\frac{n\pi}{\lambda})^2+\beta\varphi_n(t)=0$$

If we assume that the periodic force varies according to the law, then the last equation is presented in the form:

$$\frac{d^2\varphi_n}{dt^2}+\varpi_n^2(1-\frac{\mathrm{P}_1\cos\upsilon t}{\mathrm{P}_n^*})\varphi_n(t)=0 \qquad (15)$$

where indicated:

$$\varpi_n^2=\frac{1}{m}\left[KI(1+\frac{\alpha}{2})\frac{n^4\pi^4}{\lambda^4}+\beta\right], \qquad \mathrm{P}_n^*=\frac{n^2\pi^2KI(1+\frac{\alpha}{2})}{\lambda^2}+\frac{\beta\lambda^2}{n^2\pi^2} \qquad (16)$$

Omitting the indices, we present equation (15) in the form:

$$\varphi^n+\varpi^2(1-2\eta_1\cos\upsilon t)\varphi=o\ , \qquad (\eta_1=\frac{\mathrm{P}_1}{2\mathrm{P}_{kp}^*}) \qquad (17)$$

which is called Mathieu's equation.

Proceeding as in work {1}, we present a method for determining the boundaries of the regions of instability of solutions to equation (17). Determining the boundaries of instability regions comes down to finding the conditions under which a given differential equation has periodic solutions with periods T and 2T.

We are looking for a periodic solution to equation (17) with a period of 2T in the form:

$$\varphi^n+\varpi^2(1-2\eta_1\cos\upsilon t)\varphi=o\ , \qquad (\eta_1=\frac{\mathrm{P}_1}{2\mathrm{P}_{kp}^*}) \qquad (18)$$

Substituting (18) into (17) and equating the coefficients for the same $\sin\frac{n\upsilon t}{2}, \cos\frac{n\upsilon t}{2}$ we obtain the following systems of linear homogeneous algebraic equations for and:

a_n и b_n :

$$(1+\eta_1-\frac{\upsilon^2}{4\varpi^2})a_1-\eta_1a_3=0\ , \quad (1-\frac{n^2\upsilon^2}{4\varpi^2})a_n-\eta_1(a_{n-2}+a_{n+2})=0\ , \quad n=3,5,7...$$

$$(1-\eta_1-\frac{\upsilon^2}{4\varpi^2})b_1-\eta_1b_3=0\ , \quad 1-\frac{n^2\upsilon^2}{4\varpi^2})b_n-\eta_1(b_{n-2}+b_{n+2})=0\ , \quad n=3,5,7... \qquad (19)$$

The condition for the existence of periodic solutions of equation (17) is that the determinants of the resulting homogeneous systems are equal to zero (19). Combining these conditions under the sign $\pm$, we obtain the following equation:

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$$\begin{vmatrix} 1 \pm \eta_1 - \frac{\upsilon^2}{4\varpi^2} & -\eta_1 & 0... \\ -\eta_1 & 1 - \frac{9\upsilon^2}{4\varpi^2} & -\eta_1... \\ 0 & -\eta_1 & 1 - \frac{25\upsilon^2}{4\varpi^2} \end{vmatrix} = 0 \qquad (20)$$

The resulting equation connecting the frequencies of the external load with the natural frequency of the rod and the magnitude of the longitudinal force is called the equation of critical frequencies.

Similar to the case considered, we look for a periodic solution to equation (17) with a period in the form of the following series:

$$\varphi(t) = b_0 + \sum_{n=2,4,6}^{\infty} (a_n \sin\frac{n\upsilon t}{2} + b_n \cos\frac{n\upsilon t}{2} \qquad (21)$$

Substituting series (21) into (17) and proceeding in a similar way, we obtain the following systems of linear algebraic equations for the coefficients and:

$$(1 - \frac{\upsilon^2}{\varpi^2})a_2 - \eta_1 a_4 = 0 \ , \quad (1 - \frac{n^2\upsilon^2}{4\varpi^2})a_n - \eta_1(a_{n-2} + a_{n+2}) = 0 \ , \quad n = 4,6,...$$

$$b_0 - \eta_1 b_2 = 0, \quad (1 - \frac{\upsilon^2}{\varpi^2})b_2 - \eta_1(2b_0 + b_4) = 0, \qquad (22)$$

,

Equating the determinants of the resulting homogeneous equations to zero, we obtain the following equations for determining the critical frequencies:

$$\begin{vmatrix} 1 - \frac{\upsilon^2}{\varpi^2} & -\eta_1 & 0... \\ -\eta_1 & 1 - \frac{4\upsilon^2}{\varpi^2} & -\eta_1... \\ 0 & -\eta_1 & 1 - \frac{16\upsilon^2}{\varpi^2}... \end{vmatrix} = 0 \qquad (23)$$

and equation

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$$\begin{vmatrix} 1 & -\eta_1 & 0 & 0\ldots \\ -2\eta_1 & 1-\dfrac{\upsilon^2}{\varpi^2} & -\eta_1 & 0\ldots \\ 0 & -\eta_1 & 1-\dfrac{4\upsilon^2}{\varpi^2} & -\eta_1\ldots \\ 0 & 0 & -\eta_1 & 1-\dfrac{16\upsilon^2}{\varpi^2}\ldots \end{vmatrix} = 0 \qquad (24)$$

In accordance with the number n included in (18) and (21), the first, second, third, etc. are distinguished. areas of dynamic instability. In this case, the first region is called the main region of dynamic instability.

To determine the boundary of the main instability region, consider equation (20). By holding here the first diagonal element i.e. “first-order determinant” and equating it to zero we get:

$$1 \pm \eta_1 - \frac{\upsilon^2}{4\varpi^2} = 0 \qquad (25)$$

Let's transform this equation to the form:

$$\upsilon_* = 2\varpi\sqrt{1 \pm \eta_1} \qquad (26)$$

where and are determined from (16) and (17).

To determine the boundaries of the second instability region, equations (23) and (24) must be used. Restricting ourselves to second-order determinants, we will have:

$$\begin{vmatrix} 1-\dfrac{\upsilon^2}{\varpi^2} & -\eta_1 \\ -\eta_1 & 1-\dfrac{4\upsilon^2}{\varpi^2} \end{vmatrix} = 0,$$

$$\begin{vmatrix} 1 & -\eta_1 \\ -2\eta_1 & 1-\dfrac{\upsilon^2}{\varpi^2} \end{vmatrix} = 0$$

From this we obtain the following approximate formulas for critical frequencies:

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$$\upsilon_* = \varpi\sqrt{1+\frac{1}{3}\eta_1^2} \qquad \upsilon_* = \varpi\sqrt{1-2\eta_1^2} \tag{27}$$

To determine the boundary of the third region of instability, we must use equation (20). Based on the second-order determinant, we have:

$$\upsilon_* = \frac{2}{3}\varpi\sqrt{1-\frac{9\eta_1^2}{8\pm 9\eta_1^2}} \tag{28}$$

On the basis of (26)-(28), the boundaries of the regions of dynamic instability of inhomogeneous rods located on an elastic foundation are determined.

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