

## Modelling the Earth's Magnetosphere Flow Around by the Solar Wind in Kinematic Approximation

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**Abstract.** The magnetopause represents a magnetic boundary layer of the Earth. Change of its parameters determines the solar wind plasma energy supplying process for the internal structures of the magnetosphere. As far as it is impossible to solve generally the problem of MHD interaction between the solar wind and the geomagnetic field, the process of the magnetosphere flow around has been viewed in gasodynamic approximation, for which different kinematic models have been used. The most popular among them is the Parker's kinematic model, by which, for instance, distributions of magnetic and electric fields onto the magnetopause were received. However, based on comparative analysis, a conclusion has been drawn that the use Gratton's kinematic model promises to bring much more prospective results, than Parker's model. The reason for such conclusion is that the modification of Gratton's model includes the coefficient of magnetic viscosity and more realistically describes the plasma flow structure near the critical point of magnetosphere.

**Keywords:** magnetosphere, magnetosheath, magnetopause, solar wind.

The Earth's magnetosphere is a giant basin of space plasma. On the day-side of magnetosphere the magnetohydrodynamic (MHD) interaction of the Earth's magnetic (geomagnetic) field and the solar wind plasma takes place. The penetration of solar wind particles into the magnetosphere occurs generally from so-called focal area and polar cusps. In those space structures specific kinetic and MHD effects are formed, which promote the generation of large scale electric field accelerating the low-energy charged particles. This process has impulse character, and it proceeds on the background so-called quasi-viscous interaction between the solar wind and geomagnetic field. The existence of magnetosphere is of a vital importance for the development of the organic matter on Earth, because geomagnetic field protects the living organisms from the excessive solar radiation. However, certain amount of energy of the electromagnetic radiation from the Sun and the corpuscular flow from its photosphere (solar wind) still can penetrate inside the magnetosphere, which is used for the regulation of the various intra magnetospheric processes,. The central (focal) area of the magnetosheath (space between bow shock and magnetosphere) on the day side of the magnetosphere has a great importance from the point of view of the energy supply of the magnetosphere, together with the plasmosphere on night side and the polar cusps as well. The magnetopause which represents the magnetic boundary layer of the Earth, develops along the entire boundary of the magnetosphere. Exactly in this structure the effects of quasi viscous interaction between the geomagnetic field and the solar wind takes place. Also in certain cases

the process of reconnection (merging) of the force lines of the Interplanetary Magnetic Field (IMF) with the force lines of the geomagnetic field takes place. Therefore, the MHD parameters of the magnetopause are particularly variable, because they depend on the regime of the solar wind plasma flow in the magnetosheath and on the intensity and the direction of the IMF as well. The IMF factor has the great importance on the development of strong sporadic geomagnetic disturbance, global geomagnetic storms, and also on the generation of the following those Ultra Low Frequency electromagnetic waves (ULF), which takes place in the main radiation belt of Earth, plasmosphere. All this reflects on the space weather, which forms the natural electromagnetic background on the Earth and which alike the meteorological weather represents the most significant element of the vital environment.

If the magnetized body is surrounded with the stream of the conductive medium, the magnetic viscosity fulfils the function of the ordinary viscosity in some conditions. However, in the MHD approximation, when  $Pr_m = 1$  (Prandtl magnetic number), the viscous and magnetic boundary layers formed on the surface of the magnetized body coincide. When  $Pr_m \gg 1$ , accumulation of the magnetic field is possible in area the size of which significantly exceeds the thickness of the viscous layer. In these conditions, particularly efficient “substitution” of the ordinary viscosity with the magnetic viscosity becomes possible, which will be reflected on the structure of the flow of the conductive medium. In the rest part of magnetosheath solar wind moves like the ordinary ideal medium. It means that dissipation is not developed in plasma, except the space nearby the boundary of magnetosphere, where effect of magnetic viscose becomes significant. At this area MHD boundary layer is developed, which we identify with magnetopause. Due to dipole nature of Earth’s magnetic field, the meridian cross section of magnetosphere is the subject of particular important for study. Today the MHD theory of magnetopause is already formulated. however it has the certain gaps. In particular, there is a problem of the general analytical solution of the of the MHD interaction of the solar wind and the geomagnetic field. Because of that, to get the dynamic picture of the main parameters, such as the thickness and the distribution of magnetic field of the magnetopause, is possible only if the apriority is given of the regime of the solar wind flow in the magnetosheath, which can be done with the one of the kinematic models [1]. It was considered for a certain time that, among those models the most effective was Parker’s popular model for the ideal (non dissipative) non compressible medium. However, ideal electrical conductivity or zero magnetic viscosity for any plasma medium can be considered as abstraction.

Let us introduce the Cartesian system with the center at the critical point of magnetosphere. Let the  $x$ -axis be directed to the Sun and the  $y$ -axis along the boundary force line of the Earth’s magnetic field. For modeling of noncompressible plasma flow nearby critical point of magnetosphere we use the two-dimensional Parker’s kinematic model [1], effectiveness of which has been recently once again ascertained by computation experiment [2]. In Parker’s model the components of

the hydrodynamic velocity of the medium in the vicinity of the critical point of the streamlined surface is linearly dependent on the coordinates

$$u = -\alpha x, \quad v = \alpha y,$$

(1)

where  $\alpha = V_0/\lambda_0$ ,  $V_0$ -is the characteristic velocity of the solar wind before its interaction with the magnetosphere,  $\lambda_0$ -characteristic linear scale of magnetosphere's boundary.

In the plane case the model (1) can be used together with analytical model of the impulse changes in time of the magnetic viscosity of the solar wind in the magnetosheath. It is seemed that, in the interplanetary space, the solar wind plasma has infinite electrical and thermal conductivity and zero ordinary viscosity. When the solar wind passes the bow shock may occur due to the activation of various plasma mechanisms. Among these mechanisms, there are ion-acoustic and ion-cyclotron instabilities, which may induce the anomalous impulsive resistivity of the plasma. In order to take into account impulsive character of changing of magnetic viscosity coefficient  $\lambda_m = c^2 / 4\pi\sigma$  ( $c$ -velocity of light,  $\sigma$ -the electric conductivity of the solar wind) in time, the following models are offered (assuming that magnetic viscosity is scalar parameter in magnetosheath) [3]

$$1) \lambda_m = \lambda_{0m} e^{-\frac{t}{\tau_0}}; \quad 2) \lambda_m = \lambda_{0m} (1 - e^{-\frac{t}{\tau_0}}),$$

(2)

where  $\lambda_{0m}$  - is characteristic value of the magnetic viscosity coefficient,  $\tau_0$  - characteristic time of impulsive changes. It is obvious that both models have the same nature and they correspond to change of plasma electric conductivity from finite to ideal and vice versa. This gives an opportunity to obtain the approximate model nonstationary picture of the distribution of the magnetic field induction on the magnetopause.

Impulsive change of solar wind magnetic viscosity causes non stationary changes of magnetosphere's magnetic boundary layer parameters too. Target of [3] was to define the thickness  $\delta$  of magnetic boundary layer and distributions of the magnetic induction and intensity electric field analytically. For this it is useful to use Shwec's method of successive approximation, according to which we consider that dipole type geomagnetic field is constant on the boundary of magnetosphere and successively decreases across the magnetic boundary layer (magnetopause).

Thus, by using of Shwec's method it is possible to receive the topological picture of distribution of magnetic field in Jigulov's type boundary layer, which corresponds to magnetosphere's meridian cross section, using the plane equation for single component of magnetic induction

$$\frac{\partial H_y}{\partial t} + u \frac{\partial H_y}{\partial x} + v \frac{\partial H_y}{\partial y} - H_y \frac{\partial v}{\partial y} = \lambda_m \frac{\partial^2 H_y}{\partial x^2}. \quad (3)$$

For equation (3) there are following boundary conditions

$$H_y = H_0, \text{ when } x = 0; \quad H_y = 0, \text{ when } x = \delta.$$

(4)

The solution of equation (3) is the sum of two terms:  $H_y = H_{1y} + H_{2y}$ , where  $H_{1y}$  is solution of homogeneous equation

$$\frac{\partial^2 H_{1y}}{\partial x^2} = 0, \quad (5)$$

and satisfied boundary conditions (4):  $H_{1y} = H_0(1 - x/\delta)$ .

The second approximation for magnetic field is defined from equation (1) by means of  $H_{1y}$  in left side. After integrations  $H_{2y}$  satisfied homogeneous boundary conditions

$$H_{2y} = 0, \text{ when } x = 0; \quad H_{2y} = 0, \text{ when } x = \delta.$$

(6)

According to Shwec's method the following distributions are obtained, which corresponds to each model of magnetic viscosity ( $\delta'$  means time differentiation)

$$1) H_y = H_0 \left(1 - \frac{x}{\delta}\right) + H_0 \lambda_{0m}^{-1} e^{\frac{t}{\tau_0}} \left[ \left( \frac{\delta'}{\delta^2} \frac{x^3}{6} - \frac{\delta'}{6} x \right) + \alpha \left( \frac{x^3}{3\delta} - \frac{x^2}{2} + \frac{\delta}{2} x \right) \right]; \quad (7)$$

$$2) H_y = H_0 \left(1 - \frac{x}{\delta}\right) + H_0 \lambda_{0m}^{-1} \left(1 - e^{-\frac{t}{\tau_0}}\right)^{-1} \left[ \left( \frac{\delta'}{\delta^2} \frac{x^3}{6} - \frac{\delta'}{6} x \right) + \alpha \left( \frac{x^3}{3\delta} - \frac{x^2}{2} + \frac{\delta}{6} x \right) \right].$$

(8)

The thickness of magnetic boundary layer is determined from the additional condition  $\left(\frac{\partial H_y}{\partial x}\right)_{x=\delta} = 0$ . This means that large scale surface electric current, locating geomagnetic field, no longer exists on upper boundary of magnetopause

$$1) \delta^2 = 6\lambda_{0m} \alpha^{-1} \left[ e^{-\alpha t} + \left(1 - \frac{1}{\alpha\tau_0}\right)^{-1} \left( e^{\frac{t}{\tau_0}} - e^{-\alpha t} \right) \right]; \quad (9)$$

$$2) \delta^2 = 6\lambda_{0m} \alpha^{-1} \left[ \left(1 - e^{-\alpha t}\right) + \left(1 - \frac{1}{\alpha\tau_0}\right)^{-1} \left( e^{\frac{t}{\tau_0}} - e^{-\alpha t} \right) \right]. \quad (10)$$

According to (9)  $\delta_0 = (6\lambda_{0m} \alpha^{-1})^{1/2}$ , when  $t = 0$ . So, in case model 1) Earth's magnetic boundary layer a priori has a certain thickness, as opposed to case 2), for which  $\delta_0 = 0$  when  $t = 0$ .

It is important that in stationary case the thickness of hydrodynamic boundary layer is constant when flow near critical point is given by Parker's kinematic model.

Such specific feature of this model, being true for stationary magnetic boundary layer as well, results from linearly link between moving medium velocity components and coordinates [3]. Therefore, in nonstationary case the thickness of magnetic boundary layer depends only on time.

After determination of magnetic induction it is necessary to find topological picture of distribution of large scale electric field generated on magnetopause. For this Maxwell's equation can be applied

$$\operatorname{rot} \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}.$$

(11)

There exists another way as well, namely, if in Omm's law electric current density  $\vec{j}$  is determined by means of Maxwell's another equation

$$\vec{j} = \frac{c}{4\pi} \operatorname{rot} \vec{H} = \sigma \left\{ \vec{E} + \frac{1}{c} [\vec{V} \cdot \vec{H}] \right\}. \quad (12)$$

According to assumption electric field, like magnetic field, has only single component on the magnetopause. This component is directed along global surface DCF-electric current in magnetosphere's equatorial plane. So, for first variant from (11) it will be

$$E_z = -\frac{1}{c} \int_0^x \frac{\partial H_y}{\partial t} dx.$$

(13)

Whereby, for instance, for magnetic viscosity model 2) the electromagnetic drift velocity in the magnetic boundary layer  $V_d = cE_z/H_0$  may be determined from the expression

$$\begin{aligned} \frac{cE_z}{H_0} = & -\frac{\delta'}{\delta^2} \frac{x^2}{2} + \lambda_{0m}^{-1} [1 - \exp(-t/\tau_0)]^{-1} \left[ \left( \frac{\delta'' x^2}{12} - \frac{\delta'' \delta - 2(\delta')^2}{24\delta^3} x^4 \right) + \alpha \left( \frac{\delta'}{12\delta^2} x^4 - \frac{\delta'}{12} x^2 \right) \right] + \\ & + (\lambda_{0m} \tau_0)^{-1} \exp(-t/\tau_0) (1 - \exp(-t/\tau_0))^{-2} \left[ \left( \frac{\delta' x^2}{12} - \frac{\delta' x^4}{24\delta^2} \right) + \alpha \left( \frac{x^3}{6} - \frac{x^4}{12\delta} - \frac{\delta}{12} x^2 \right) \right]. \end{aligned} \quad (14)$$

The accuracy of this analytical model hesitates within the 15-20%, which is according to the latest numerical (computer simulations) experiments is sufficient for the satisfaction of the criteria of the validity of the single liquid MHD approximation near the boundary of the magnetosphere. However, the Parker's kinematics model in case of rigorous discussion is true only at a small distance from the critical (focal) point of magnetosphere. Hence, it is necessary to expand the modeling area of the solar wind flow, which has to include the significant part of the magnetosheath from the bow shock to the critical point of magnetosphere. It is evident that the effect of solar wind compressibility must show up both in a large-scale MHD pattern of the flow around the magnetosphere and in those cinematic effects which cause the change in an electrical conductivity of plasma. Therefore, it is necessary to use more

efficient kinematics models than the Parker's one. We consider Gratton's kinematic model or it's some modification for the compressible viscose medium as a relevant, because it give us the hydrodynamic picture of plasma flow at any distance from the critical point of magnetosphere [4]. In contrast to Parker's model which hold for the ideal plasma, Gratton's model requires that the medium should have normal or equivalent magnetic viscosity. In two dimensional form for components of velocity of compressible magnetized plasma of solar wind according this model we have [5]

$$V_x = -V_0 \left( 1 - e^{-\frac{V_0 x}{\lambda_m}} \right), \quad V_y = \beta \frac{V_0^2}{\lambda_m} y e^{-\frac{V_0 x}{\lambda_m}}, \quad (15)$$

where again the Cartesian system with center at the critical point of magnetosphere is used.  $\beta$  - the numerical coefficient of compressibility.

The kinetic theory supposes that near the day side boundary of the magnetosphere, the essential conditions for development of anomalous resistivity may be established due to the formation of plasma instability, the abrupt changes in electromagnetic characteristics of the plasma medium, and the formation of discontinuities and, also, of local double-potential electrical layers. The basic characteristics of plasma such as electric current density or Alven and thermal velocities, like transport coefficients, are related to the density of the plasma and determines not only gasodynamic but more complex magnetohydrodynamic patterns of the flow around the magnetosphere. As a result, in the vicinity of the magnetosphere boundary, the solar wind develops the magnetic viscosity and therefore the use of the Gratton's model is a relevant for the theoretical calculations and the analytical interpretation of the results of the numerical experiment. It is obvious, that the Gratton's model modification (15) is valid for the medium which has the magnetic viscosity. Precisely this is the main distinctive sign of the Gratton's model compare to Parker's model, which is valid only for the ideally conductive and nonviscous noncompressible medium. As positive example, my be mention results of [5], where the simple theoretical model of plasma density variation near the critical point of the magnetosphere in gasodynamic approximation is constructed. The computer experiment have shown that the plasma depletion layer is certainly formed at the magnetosphere boundary. But it is impossible to define the analytical pattern of the layer parameters by numerical calculations. But the theoretical analysis shows that the density is zero at the stagnation point, but nearby the surface is formed a plasma depletion layer which, moving from the stagnation point, recedes from the surface. Besides the decrease in density of depletion layer, similiary to the numerical experiment, there occurs compression of the plasma to the magnetosphere's periphery. Such effect will take place in the model problem only of plasma velocity at the flanks of magnetosheath exceeds the value of the solar wind velocity which is used in the modified Gratton's model. But it should be noted that the comparison with the numerical experiment does not answer the question whether the effect of moving-away of the depletion layer from the surface is physically correct or not. This

problem may be associated with the plasticity of the magnetosphere boundary, taking into account the curvature of the surface flown around. Since the problem of flow around the magnetosphere is still actual, it is not improbable that the issue of curvature of geomagnetic field boundary force lines will become the focus of attention in the immediate future.

In MHD case only one study using the Gratton's model is known, which results are more or less sufficiently informative [4]. So, in case of magnetic boundary layer, unlike a dynamic boundary layer problem, where the Gratton's model already confirmed its efficiency, it is necessary to conduct the specific research. But we have optimism, that it is possible to obtain the effective analytical solution of the complicated version of the induction equation, if more adroit analytic method than in [3] will be used, even if it would be approximate. In such a case, it may be possible to correctly evaluate the effectiveness of Gratton's model application in MHD case. For instance, for this may be used Shwec's method of successive approximation or the VKB approximate analytical method. It is possible, that the new results successfully will be used for the estimation of the compliance of the theoretical modeling and the results of numerical experiments. So, we are optimistic that using the Gratton's model will be possible to receive adequate topological picture of magnetic field and obtained the interval of the changes of the value of the velocity of the electromagnetic drift. That is very important for diagnostic of the MHD effects of couplings of solar wind and magnetosphere. Also the threshold of the development of the effect of the anomalous resistivity in the plasma of the solar wind will be determined. The value of the magnetic viscosity and the criterions of the electromagnetic waves generation caused by the various plasma instabilities are dependent on this determination..

## References

1. Aburdjania G.D., Kereselidze Z.A., Khantadze A.G., Chkitunidze M.S. Large-Scale LF Electromagnetic Waves in the Earth's Magnetosheath. *Geomagnetism and Aeronomy* 2007, Vol.47, #5. pp.548-554.
2. Dorelli J.C., Hesse M., Kuznetsova M.M., Rastaetter L. A New Look at Driven Magnetic Reconnection at the Terrestrial Subsolar Magnetopause. *Journal of Geophys. Research*, 2004, vol.109, A 12216, doi:10.1029/2004JA010458.
3. Kereselidze Z., Chkhitunidze M., Kirtskhalia V., Kalandadze I. On Modeling of Magnetic Boundary Layer on the Dayside Magnetosphere. *Georgian International J. of Sci. and Tex.*, 2008, v.1, #3, pp. 41-47.

4. Gratton F.T., Gnani G, Heyn M.F, Biernat H.K.,Rijnbeek R.P., Pressure Drive and Viscous Dragging: A Reply, J. Geophys. Res., 1990, Vol. 95, No A1, p.261-163.

5. Kereselidze Z.A., Ghurtskaia N.V., Kurashvili G.A. A Mode of the Plasma Density Variation Near the Critical Point of the Earth's Magnetosphere. Georgian Engineering news, 2005, #1, pp. 26-32.